Buckling of Composite Plates with a Free Edge in Edgewise Bending and Compression

James Ting-Shun Wang*
Georgia Institute of Technology, Atlanta, Georgia
and
Sherrill B. Biggers† and John N. Dickson‡
Lockheed-Georgia Company, Marietta, Georgia

Buckling of rectangular plates with the two loaded edges simply supported, one side free and the other side either simply supported, clamped, or rotationally restrained by an elastic support, is investigated for linearly varying compressive load. An analysis procedure applicable to orthotropic laminated plates is presented. The Galerkin procedure with Legendre polynomials as shape functions is used. Inasmuch as the set of Legendre polynomials is complete, the convergence of solutions is assured. The well-understood properties of Legendre polynomials allow explicit expressions for the elements of the buckling coefficient matrix to be obtained with ease. As a result, these elements can be efficiently generated with a systematic computational procedure. Some numerical examples are included for illustrative purposes. Results for isotropic plates, presented in a graphical form suitable as design charts, are also presented.

Nomenclature

a	= plate length
b	= plate width
D_{11}, D_{12}, D_{22}, D_{66}	= plate bending stiffnesses
$k_r, \bar{k}_r$	= rotational stiffness of the support
$k, \bar{k}$	= buckling coefficient
L	= linear differential operator
N_x	= longitudinal stress resultant
x, y	= Cartesian coordinates
α	= load variation parameter

Introduction

BUCKLING of uniformly compressed rectangular plates has been studied extensively. Results may be found in standard textbooks[1-3] as well as numerous technical articles. The buckling analysis for rectangular plates simply supported on all sides and subjected to a linearly varying load representing combined bending and compression can be found in Ref. 1 for isotropic materials and in Ref. 4 for sandwich plates. Walker[5] investigated the buckling of isotropic rectangular plates with various supporting conditions along the unloaded edges by the Galerkin method. Simple polynomials satisfying the boundary conditions along the unloaded edges are used as shape functions. With these selected shape functions the procedure does not lead to explicit expressions for the elements of the buckling coefficient matrix. Buckling analyses of rectangular plates subject to uniformly distributed loads have been reported in Ref. 2 for anisotropic plates and in Ref. 6 for isotropic plates. While limited studies on isotropic plates under linearly varying axial loading have been presented, an efficient analysis procedure for orthotropic plates with one edge free is not immediately available in the open literature. This class of plates occurs frequently in aircraft structures, such as flanges of beams and stiffeners on stiffened plates loaded in compression and/or weak-axis bending.

The purpose of this study is to present an analysis for determining the buckling load of nonuniformly compressed rectangular composite plates with the two simply supported loaded edges, one side free and the other side either simply supported, clamped, or rotationally restrained by an elastic support. A sinusoidal buckling mode in the longitudinal direction is assumed. The mode shape in the width direction is represented by a complete set of orthogonal polynomials and the extended Galerkin procedure is used. Solutions for the physically realistic case with linearly varying compressive load parallel to the longitudinal direction are presented in detail. Numerical examples are included for illustrative purposes and to demonstrate the efficiency and accuracy of the procedure.

Analysis

A rectangular plate having one free edge and subjected to a linearly varying load as shown in Fig. 1 is considered. The extended Galerkin procedure is used to determine the buckling load. The length a represents the half-wavelength of the buckling mode along the longitudinal direction and b is the width of the plate. The shape functions used in the Galerkin procedure satisfy the simply supported boundary conditions along $\bar{x}=0$ and 1 where $\bar{x}=x/a$. Likewise, the geometrical boundary condition is satisfied along $\xi=y/b=0$ where an elastic rotational restraint may be specified. The linear buckling problem for orthotropic plates subject to the above conditions is governed by the following equation:

$$\begin{aligned} &\int_0^1 \int_0^1 L(w)\,\delta w \,\mathrm{d}\xi \,\mathrm{d}\bar{x} - \int_0^1 \left\{ \left[\frac{D_{22}}{b^3} \frac{\partial^3 w}{\mathrm{d}\xi^3} + \left(\frac{D_{12}+4D_{66}}{a^2 b} \right) \frac{\partial^3 w}{\partial \xi \partial \bar{x}^2} \right] a\delta w \right\}_{\xi=1} \mathrm{d}\bar{x} \\ &+ \int_0^1 \frac{a}{b} \left[\left(\frac{D_{22}}{b^2} \frac{\partial^2 w}{\partial \xi^2} + \frac{D_{21}}{a^2} \frac{\partial^2 w}{\partial \bar{x}^2} \right) \frac{\partial}{\partial \xi} (\delta w) \right]_{\xi=1} \mathrm{d}\bar{x} \\ &+ \int_0^1 \left[k_r \frac{a}{b^2} \frac{\partial w}{\partial \xi} \frac{\partial}{\partial \xi} (\delta w) \right]_{\xi=0} \mathrm{d}\bar{x} = 0 \end{aligned} \tag{1}$$

Received July 15, 1982; revision submitted June 2, 1983.

*Professor, School of Engineering Science and Mechanics; also, Consultant, Lockheed-Georgia Company, Marietta, Ga.

†Specialist Engineer, Advanced Structures Department.

‡Staff Specialist, Advanced Structures Department.

in which w is the transverse displacement and the linear differential operator L is given by

$$L=D_{11}\frac{\partial^4}{a^4\partial\bar{x}^4}+2(D_{12}+2D_{66})\frac{\partial^4}{a^2b^2\partial\xi^2\partial\bar{x}^2}+D_{22}\frac{\partial^4}{b^4\partial\xi^4}+N_x\frac{\partial^2}{a^2\partial\bar{x}^2}$$

where D_{11}, D_{12}, D_{22}, and D_{66} are plate bending stiffnesses as defined in Ref. 2 and k_r is the rotational stiffness of the support along $\xi=0$. Clearly, $k_r=0$ and ∞ correspond to simply supported and clamped edges, respectively. The general solution w is expressed in the following series form:

$$w=\sum_m B_m P_m(\xi)\sin\pi\bar{x} \tag{2}$$

in which the $P_m(\xi)$ are the shifted Legendre polynomials. Only odd integers for m are considered in the analysis. The axially applied load along $\bar{x}=0$ and 1 takes the following general form:

$$N_x=N(1-\alpha\xi) \tag{3}$$

where compression is positive. A uniform loading may be defined by setting $\alpha=0$. With $\alpha=2$, a state of pure bending is specified. Other values of α define various combinations of uniform and bending loads. Applying the extended Galerkin procedure in conjunction with the substitution of Eqs. (2) and (3) into Eq. (1), the following system of symmetric algebraic equations is obtained:

$$[a_{nm}]\{B_m\}=0 \tag{4}$$

in which

$$a_{nm}=J_{nm}+S_{nm}+F_{nm}+K_{nm}=a_{mn} \tag{5}$$

The detailed expressions for J_{nm}, S_{nm}, F_{nm}, and K_{nm} are as follows for $n\geq m$:

$$\sum_m J_{nm}B_m=\int_0^1\int_0^1 L(w)P_n(\xi)\sin\pi\bar{x}\,d\bar{x}\,d\xi \tag{6}$$

$$J_{nm}=C\left(\frac{b}{a}\pi\right)^4\left[\left(\frac{D_{11}}{D_{22}}-\frac{Na^2}{\pi^2D_{22}}\right)\frac{1}{2m+1}\delta_{mn}-\frac{\alpha Na^2}{\pi^2D_{22}}I_{nm}\right] \tag{7}$$

$$C=D_{22}a/2b^3$$

$$N^*=(Nb^2/\pi^2D_{22})(a/b)^2$$

$$I_{nm}=\int_0^1\xi P_m(\xi)P_n(\xi)\,d\xi=\frac{1}{2m+1}P_n'(0)\left[\frac{(m+1)P_{m+1}(0)}{(n-m-1)(n+m+2)}+\frac{mP_{m-1}(0)}{(n-m+1)(n+m)}\right]$$

$$\sum_m S_{nm}B_m=-\int_0^1\left[\left(\frac{D_{22}}{b^3}\frac{\partial^3w}{\partial\xi^3}+\frac{D_{12}+4D_{66}}{a^2b}\frac{\partial^3w}{\partial\bar{x}^2\partial\xi}\right)a\delta w\right]_{\xi=1}d\bar{x} \tag{8}$$

$$S_{nm}=-C\left[P_m'''(1)-\left(\frac{\pi b}{a}\right)^2\frac{D_{12}+4D_{66}}{D_{22}}P_m'(1)\right]P_n(1) \tag{9}$$

$$\sum_m F_{nm}B_m=\int_0^1\left[\left(\frac{D_{22}}{b^2}\frac{\partial^2w}{\partial\xi^2}+\frac{D_{21}}{a^2}\frac{\partial^2w}{\partial\bar{x}^2}\right)\frac{a}{b}\frac{\partial}{\partial\xi}(\delta w)\right]_{\xi=1}d\bar{x} \tag{10}$$

$$F_{nm}=C\left[P_m''(1)-\left(\frac{\pi b}{a}\right)^2\frac{D_{21}}{D_{22}}P_m(1)\right]P_n'(1) \tag{11}$$

$$\sum_m K_{nm}B_m=\int_0^1\left[k_r\frac{a}{b^2}\frac{\partial w}{\partial\xi}\frac{\partial}{\partial\xi}(\delta w)\right]_{\xi=0}d\bar{x} \tag{12}$$

$$K_{nm}=Ck_r\frac{b}{D_{22}}P_m'(0)P_n'(0) \tag{13}$$

where primes denote the derivatives of functions with respect to ξ. The elements of the coefficient matrix of Eq. (4) have been easily and explicitly determined because of the orthogonality and other well-understood properties of the Legendre polynomials. Inasmuch as the set of Legendre polynomials is complete, the convergence of the solution is assured. The rapidity of this convergence is demonstrated by several illustrative examples that follow. Furthermore, the high degree of computational efficiency that is achieved in evaluating the coefficient matrix for specific cases is evident by noting the extremely simple form of the components defined above and the fact that the matrix is symmetric.

The critical value of the load parameter N^* corresponding to a buckling condition is determined by requiring the determinant of the coefficient matrix of Eq. (4) to be equal to zero. A number of search techniques are available to locate the N^*_{cr} yielding a zero determinant. Alternatively, Eq. (4) (specifically the J_{nm} components) may be reformulated to yield a generalized eigenvalue problem enabling the use of standard techniques to obtain N^*_{cr}. The approach taken here in obtaining numerical results was to use the false position method in a determinant search. This simple procedure resulted in an extremely efficient and accurate solution routine.

A Simple Example

To illustrate the procedure and to verify the analysis as applied to a simple case with a known exact solution, the buckling load for an isotropic square plate with the plate rigidity D and Poisson's ratio ν subjected to uniform compression ($\alpha=0$) is calculated manually by taking only two terms in the assumed series solution. For an isotropic plate, $D_{11}=D_{22}=D_{12}+2D_{66}=D$, $D_{12}=\nu D$, and $D_{66}=(1-\nu)D/2$. The elements of the coefficient matrix can then be written as

$$a_{nm}=\alpha_{nm}-\beta_{nm}N^* \tag{14}$$

in which, for $\nu=0.25$,

$$\alpha_{11}=\tfrac{1}{3}\pi^4+2(1-\nu)\pi^2=47.2741$$

$$\alpha_{31}=(2-7\nu)\pi^2=2.4674=\alpha_{13}$$

$$\alpha_{33}=75+9\pi^2+(1/7)\pi^4=177.7420$$

$$\beta_{11}=\tfrac{1}{3}\pi^4=32.4697$$

$$\beta_{31}=0=\beta_{13}$$

$$\beta_{33}=(1/7)\pi^4=13.9156$$

The buckling equation becomes

$$a_{11}a_{33}-a_{31}a_{13}=(\beta_{11}\beta_{33}-\beta_{31}^2)(N^*)^2-(\alpha_{11}\beta_{33}+\alpha_{33}\beta_{11}-2\alpha_{31}\beta_{31})N^*-(\alpha_{11}\alpha_{33}-\alpha_{31}^2)=0 \tag{15}$$

which may be reduced to

$$(N^*)^2 - 14.2289(N^*) + 18.5831 = 0 \tag{16}$$

The smallest root of Eq. (16) for N^* corresponds to the buckling load

$$N_{cr} = N^*_{cr}(\pi^2 D/b^2) = 1.4547(\pi^2 D/b^2) \tag{17}$$

The exact value of N^*_{cr} given in Ref. 1 is 1.440. This simple example indicates that the analysis presented is effective in determining the buckling load for the case where the edge along $\xi = 0$ is simply supported.

Plates Clamped along $\xi = 0$

As indicated in the last simple example, solutions converge rapidly for plates simply supported along $\xi = 0$. Later examples show that the analysis is also accurate and efficient for combined compression and bending cases for plates simply supported along $\xi = 0$. While the assumed set of shape functions containing only odd functions of $P_m(\xi)$ results in rapid convergence of solutions for plates simply supported along $\xi = 0$, the convergence of the solution is found to be slower for plates elastically constrained along $\xi = 0$ and particularly so for the case where the edge is fixed. For this reason, a separate solution is presented here for plates clamped along $\xi = 0$. For this case, the transverse displacement is represented in the following form:

$$w = \sum_{m=2,4\ldots} B_m [P_m(\xi) - P_m(0)] \sin \pi \bar{x} \tag{18}$$

This expression satisfies the clamped boundary conditions along $\xi = 0$ exactly. Using Eq. (18) and following the same procedure presented for the general case, the elements of the coefficient matrix for $n \geq m$ are found to be

$$J_{nm} = C\Big\{\left(\frac{\pi b}{a}\right)^4 \Big[\left(\frac{D_{11}}{D_{22}} - N^*\right)\frac{1}{2m+1}\delta_{mn} + P_m(0)P_n(0) + \alpha N^* I_{nm}\Big] - \Big[P'''_M(1) - \frac{2(D_{12}+2D_{66})}{D_{22}}\left(\frac{\pi b}{a}\right)^2 P'_m(1)\Big]P_n(0)\Big\} \tag{19}$$

in which

$$I_{nm} = I_{nm1} + P_n(0) I_{nm2} + P_m(0) I_{nm3} + \tfrac{1}{2} P_m(0) P_n(0)$$

where

$$I_{nm1} = \int_0^1 \xi P_m(\xi) P_n(\xi)\,d\xi = \Big[\frac{(n+1)P'_{n+1}(0)}{(n-m+1)(n+m+2)} + \frac{nP'_{n-1}(0)}{(n-m-1)(n+m)}\Big]\frac{P_m(0)}{2n+1}$$

$$I_{nm2} = \int_0^1 \xi P_m(\xi)\,d\xi = -\frac{1}{2m+1}\Big\{\frac{m+1}{2m+3}[P_{m+2}(0) - P_m(0)] + \frac{m}{2m-1}[P_m(0) - P_{m-2}(0)]\Big\}$$

$$I_{nm3} = \int_0^1 \xi P_n(\xi)\,d\xi = -\frac{1}{2n+1}\Big\{\frac{n+1}{2n+3}[P_{n+2}(0) - P_n(0)] + \frac{n}{2n-1}[P_n(0) - P_{n-2}(0)]\Big\}$$

$$S_{nm} = -C\Big[P'''_m(1) - \frac{D_{12}+4D_{66}}{D_{22}}\left(\frac{\pi b}{a}\right)^2 P'_m(1)\Big][1 - P_n(0)] \tag{20}$$

$$F_{nm} = C\Big\{P''_m(1) - \left(\frac{\pi b}{a}\right)^2 \frac{D_{21}}{D_{22}}[1 - P_m(0)]\Big\}P'_n(1) \tag{21}$$

$$K_{nm} = 0$$

The coefficient matrix becomes

$$[a_{nm}] = [J_{nm} + S_{nm} + F_{nm}] = 0 \tag{22}$$

It should be noted that the above matrix is symmetric and m and n are even integers.

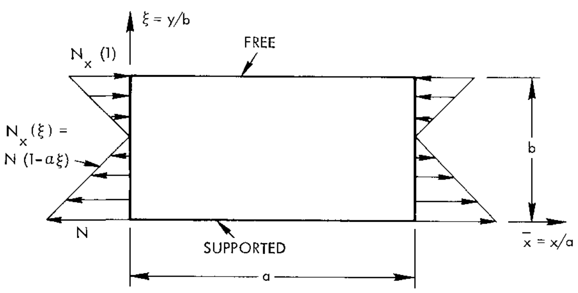

Fig. 1 Geometry and loading.

Table 1 Buckling coefficient $\bar{k}$

	Support condition at $\xi = 0$		
a/b	Simply supported	Elastically restrained ($\bar{k}_r = 4$)	Clamped
1.0	47.09	54.22	67.31
1.2	38.64	46.97	60.93
1.4	33.23	43.03	58.40
1.6	29.57	41.11	58.30
1.8	27.00	40.52	59.89
2.0	25.13	40.86	62.74
2.2	23.72	41.91	63.51
2.4	22.64	43.52	60.93
2.6	21.80	44.69	59.30
2.8	21.12	43.03	58.40
3.0	20.57	41.87	58.11

Table 2 Buckling stress resultants

	Support condition at $\xi = 0$					
	Simply supported			Clamped		
α	n	$N_x(\xi=1)$	$N_x(\xi=0)$	n	$N_x(\xi=1)$	$N_x(\xi=0)$
0.99	1	34.78	3478.0	4	75.48	7548.0
0.50	1	725.4	1451.0	3	1454.0	2908.0
0	1	908.2	908.2	3	1768.0	1768.0
−1	1	1039.0	519.4	3	1979.0	989.4
−99	1	1208.0	12.1	3	2236.0	22.4
3	1	1454.0	−727.2	3	2579.0	−1289.0
2	1	1813.0	−1813.0	3	3018.0	−3018.0
1.50	1	3456.0	−6913.0	3	4401.0	−8801.0

Numerical Examples

To provide further verification of the accuracy and efficiency of the solution procedure, buckling loads for isotropic plates ($\nu = 0.3$) with $\alpha = 1.0$ were obtained for comparison with results presented for this case in Ref. 5. The

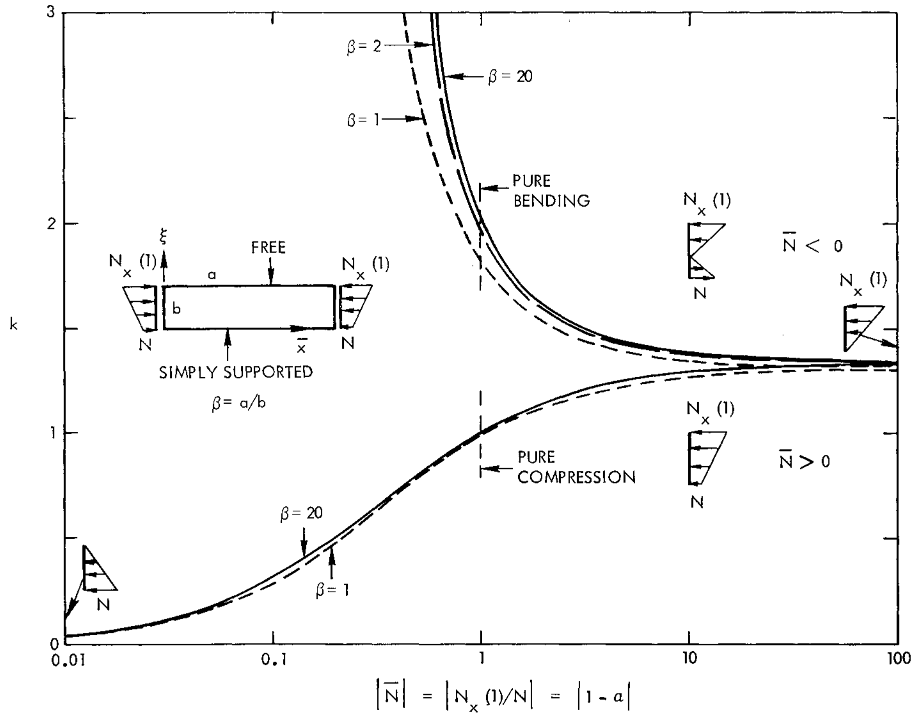

Fig. 2 Buckling coefficient vs load variation (simply supported along $\xi = 0$).

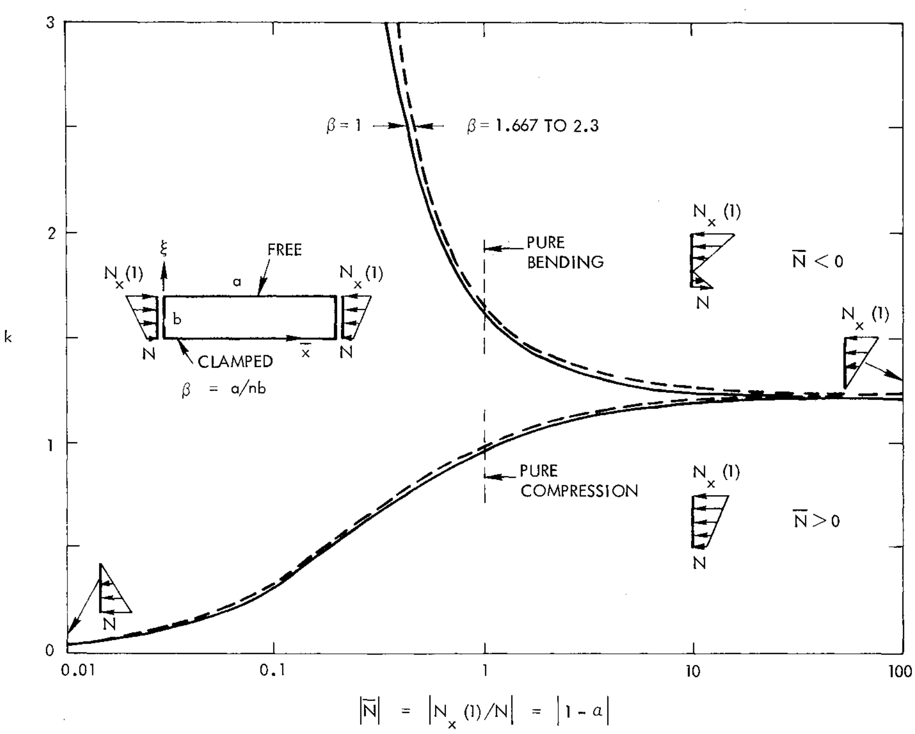

Fig. 3 Buckling coefficient vs load variation (fixed along $\xi = 0$).

loading with $\alpha = 1.0$ is triangular in shape with the nonzero magnitude at the supported edge. Buckling coefficients obtained with the procedure described here are listed in Table 1 for simply supported, elastically restrained, and clamped conditions along $\xi = 0$. The buckling coefficient is defined as in Ref. 5,

$$\bar{k} = Nb^2/D = \pi^2 N^* (b/a)^2 \tag{23}$$

and the elastic restraint parameter is defined as

$$\bar{k}_r = bk_r/D \tag{24}$$

To study the convergence of the solution, results were obtained by successively increasing the number of terms included in the series expressions until no difference to four significant figures occurred in two successive buckling coefficients. This convergence required 5, 22, and 16 terms for the simple, elastic, and clamped support conditions, respectively. Results within 1% of these converged values were obtained with 3, 10, and 6 terms for the respective support conditions. The converged values listed in Table 1 are approximately 1-3% lower than those given in Fig. 6 of Ref. 5. From these results it appears that no more than 10-12 terms would be required in most practical applications. These results also show the importance of considering the rotational restraint along $\xi = 0$ that may be provided to the free-edged plate by, for example, the skin of a blade-stiffened panel or the web of an I-beam. The benefit of this rotational restraint increases as the aspect ratio a/b increases.

As a second example, consider a composite plate with a $[\pm45_3/0_3/90_3]_s$ layup of a typical graphite/epoxy material. The bending stiffness matrix with $D_{11}=46.32$ N·m (409.94 in.-lb) is

$$[D]=\begin{bmatrix} 1 & 0.7097 & 0 \\ 0.7097 & 0.9050 & 0 \\ 0 & 0 & 0.7112 \end{bmatrix} D11 \tag{25}$$

where the small D_{16} and D_{26} values are set equal to zero. With length of 254 mm (10 in.) and width of 50.8 mm (2 in.), the results for the buckling stress resultants N_x at $\xi=0$ and $\xi=1$ corresponding to various values of α are given in Table 2 for both simply supported and clamped along $\xi=0$. It should be noted that the negative values of N_x shown in Table 2 are tensile and $(1-\alpha)$ represents the ratio of N_x at $\xi=1$ to that at $\xi=0$. The values of n shown in Table 2 are the critical number of longitudinal half-waves for this example. Simply supported plates will always be critical in the longest possible wavelength. Rotationally restrained or clamped plates may be critical in shorter wavelengths, depending on the degree of restraint, the aspect ratio, the relative values of the plate stiffnesses, and the load variation. The values shown in Table 2 were obtained using 12 terms in the series defined by Eq. (2).

The benefit of rotational restraint along the edge $\xi=0$ may again be observed from the results in Table 2. Infinite restraint increases the critical loads from 27% ($\alpha=1.5$) to 117% ($\alpha=0.99$) over the simply supported critical loads for this example. Finite rotational restraint results in smaller increases.

Data similar to that in Table 2 but for an isotropic material ($\nu=0.33$) are shown in graphical form in Figs. 2 and 3. Here the buckling coefficient k defined by

$$N_{cr}\,|_{\xi=1}=kN_0 \tag{26}$$

is shown as a function of the absolute value of $\bar{N}$, which is the ratio of N_x at $\xi=1$ to that at $\xi=0$ or $|1-\alpha|$. The multiplier N_0 is an approximate buckling load with uniform distribution ($\alpha=0$ or $\bar{N}=1$). For a plate simply supported along $\xi=0$, N_0 is given by

$$N_0=(1/b^2)\,[12D_{66}+\pi^2D_{11}/\beta^2] \tag{27}$$

in which $\beta=a/b$ and the single-term displacement function

$$w=A\xi\sin\pi\bar{x} \tag{28}$$

was used in an energy solution. For a plate fixed along $\xi=0$, a similar approximate solution is

$$N_0=(1/b^2)\,[\,(\pi^2D_{11}/\beta^2)+17.8D_{66}-1.48D_{12}+1.26D_{22}\beta^2\,] \tag{29}$$

when the single-term displacement function

$$w=A\xi^2\,(6-4\xi+\xi^2)\sin\pi\bar{x} \tag{30}$$

is used. In the latter case, $\beta=a/(nb)$ and $\bar{x}=nx/a$, where n is the critical integer number of longitudinal half-waves. For $\alpha=0$, minimum N_0 corresponds to one of the integer values of n adjoining

$$\beta=1.672(D_{11}/D_{22})^{1/4} \tag{31}$$

The approximate buckling loads given by Eqs. (27) and (29) are accurate to within 4% as shown by the values of k for $\bar{N}=1.0$ in Figs. 2 and 3. Similar graphs of k could be prepared for plates of other isotropic materials or for laminated plates. This form of the data is well suited as design charts since the effect of plate dimensions is minimized and the interpolation error for arbitrary plate sizes is therefore minimized.

Figure 2 shows results for plates simply supported along $\xi=0$. The k values are not strongly dependent on aspect ratio except for β less than 2 and $\bar{N}$ less than zero. In this case, the plate dimensions define β since the critical wavelength always equals the plate length. Figure 3 shows results for plates fixed along $\xi=0$ with $\beta=1$ and 1.667-2.3. The 1.667 value is the critical longitudinal wave aspect ratio for a very long plate in this case. Values of k for β of 1.667-2.3 are practically indistinguishable. Values of k for β of 1.0-1.667 may be interpolated with little error.

The critical value of β for fixed-edge plates may need to be determined by calculating the buckling loads for the feasible values of n on either side of the value given by Eq. (31).

Conclusions

A procedure for determining the buckling load of a rectangular composite plate having one edge free and subjected to a linearly varying edge compressive load has been developed. The extended Galerkin procedure, with Legendre polynomials as the shape functions, was used to develop simple, explicit expressions for the buckling coefficient matrix. These expressions are presented for plates with simply supported, elastically restrained, and clamped conditions along the supported longitudinal edge. Since the chosen shape functions are a complete set, convergence of the solution procedure is assured. The rapidity of the convergence as well as the accuracy and efficiency of the procedure have been demonstrated through several numerical examples. No more than 10-12 terms appear necessary in the series expressions in most practical cases. More terms may be required for highly orthotropic plates or for plates with strong elastic restraint. However, to a large extent, the simplicity of the evaluation of the buckling coefficient matrix lessens the impact on the efficiency of increasing the number of terms. In all the cases considered, regardless of the number of terms used, the computation time was inconsequential by almost any standard. The effects of load variation α and edge restraint on the buckling loads have been demonstrated through the examples. A convenient way to graphically represent buckling coefficients as functions of load variation has been presented.

References

[1]Timoshenko, S. P. and Gere, J. M., *Theory of Elastic Stability*, 2nd ed., McGraw Hill Book Co., New York, 1961.

[2]Ashton, J. E. and Whitney, J. M., *Theory of Laminated Plates*, Technomic Publishing Co., Stamford, Conn., 1969.

[3]Ugural, A. C., *Stresses in Plates and Shells*, McGraw Hill Book Co., New York, 1981.

[4]Kimel, W. R. and Raville, M. E., "Buckling of Sandwich Panels in Edgewise Bending and Compression," *Proceedings of the 4th U.S. National Congress of Applied Mechanics*, ASME, New York, 1962, pp. 657-666.

[5]Walker, A. C., "Flat Rectangular Plates Subjected to a Linearly-Varying Edge Compressive Loading," *Thin-Walled Structures*, edited by A. H. Chilver, Chatto and Windus, London, 1967, pp. 208-247.

[6]Dawe, D. J. and Roufaeil, O. L., "Buckling of Rectangular Mindlin Plates," *Computers and Structures*, Vol. 15, No. 4, 1982, pp. 461-471.